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Technical Report No. 4

UNIQUENESS OF THE CONCENTRATED-LOAD PROBLEM IN THE LINEAR THEORY OF COUPLE-STRESS ELASTICITY

by
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UNIQUENESS OF THE CONCENTRATED-LOAD PROBLEM IN THE LINEAR THEORY OF COUPLE-STRESS ELASTICITY[1]

by

R. J. Hartranft[2] and G. C. Sih[3]

In recent years, considerable attention has been focused on the linear theory of couple-stress elasticity. This Note is concerned with the development of certain conditions for uniqueness of solution in the couple-stress theory involving concentrated surface loads.

Because of the extensiveness of the literature on couple-stress problems, only those references which are relevant to the present investigation will be cited. The influence of couple-stresses on the stress distribution in a semi-infinite plane subjected to concentrated surface loads was studied by Muki and Sternberg [1][4], Tiwari [2], and Bert and Appl [3]. In [2,3], the conventional stresses were found to coincide with the Boussinesq solution of classical elasticity, while the couple-stresses were found to possess singularities of order r^{-2}, r being the radial distance measured from the point of application of the load. With the aid of Fourier transforms, Muki and Sternberg [1] solved the same problem but obtained results that disagree seriously with those in [2,3]. For the case of a concentrated load applied normal to the surface of a half-plane, one of the couple-stresses possessed merely a logarithmic singularity and the other remained finite at r = 0. In addition, the detailed structure of the singular terms of the conventional stresses is entirely different from that of the classical solution. The disagreement between the singular solutions in [1] and [2,3] could not be settled by the uniqueness theorem of Mindlin and Tiersten [4], since their theorem does not hold in the presence of discontinuous boundary loads. The need for a unique characterization of the concentrated-load problem in couple-stress elasticity is apparent.

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[4]Number in brackets designate References at end of Note.

Within the framework of the classical theory of elasticity, Sternberg and Eubanks [5] extended the classical uniqueness theorem to boundary-value problems with concentrated loads. They replaced the concentrated load by a system of statically equivalent surface tractions which are distributed continuously over a finite surface element around the concentrated-load point. The solution to the original problem is then defined as the limit of the solution to the distributed-loading problem, which is covered by the classical uniqueness theorem, as the surface element is shrunk to the load point. This limit-definition will also be adopted here in an effort to provide a unique formulation of the concentrated-load problem in the couple-stress theory of linear elasticity.

Stored Energy in Cosserat Medium

Before proceeding with the proof of uniqueness of solution in the presence of concentrated loads, the stored-energy expression will be cast into a convenient form. Under the conditions of plane strain [6], the energy density function for a Cosserat medium is

$$2W = \sigma_x \varepsilon_x + \sigma_y \varepsilon_y + (\tau_{xy}+\tau_{yx}) \varepsilon_{xy} + \mu_x \kappa_x + \mu_y \kappa_y , \qquad (1)$$

in which the curvatures κ_x, κ_y are proportional to the couple-stresses μ_x, μ_y[5]:

$$\kappa_x = \frac{1}{4\eta} \mu_x , \qquad \kappa_y = \frac{1}{4\eta} \mu_y . \qquad (2)$$

The strain and stress relationships are

$$\varepsilon_x = \frac{1}{2G}[\sigma_x - \nu(\sigma_x+\sigma_y)] , \qquad \varepsilon_y = \frac{1}{2G}[\sigma_y - \nu(\sigma_x+\sigma_y)] ,$$
$$\varepsilon_{xy} = \frac{1}{4G}(\tau_{xy}+\tau_{yx}) . \qquad (3)$$

Substitution of eqs. (2) and (3) into (1) gives

$$2W = \lambda(\varepsilon_x+\varepsilon_y)^2 + 2G(\varepsilon_x^2+\varepsilon_y^2+2\varepsilon_{xy}^2) + 4\eta(\kappa_x^2+\kappa_y^2) , \qquad (4)$$

[5] The constant η in eq. (2) corresponds to the modulus of curvature or bending, B, in Mindlin's paper [6].

where λ, G are the Lamé coefficients and they are related to Poisson's ratio ν as

$$\lambda = 2G\nu/(1-2\nu) \quad .$$

In order that W in eq. (4) is positive definite, it is necessary and sufficient to require

$$\lambda > 0 \ , \quad G > 0 \ , \quad \eta > 0 \quad .$$

Moreover, knowing that the strains are related to the displacements u_x, u_y by[6]

$$\varepsilon_x = u_{x,x} \ , \quad \varepsilon_y = u_{y,y} \ , \quad 2\varepsilon_{xy} = u_{x,y}+u_{y,x} \ , \tag{5}$$

and the curvatures to the rotation, $2\omega_z = u_{y,x}-u_{x,y}$, by

$$\varkappa_x = \omega_{z,x} \ , \quad \varkappa_y = \omega_{z,y} \tag{6}$$

eq. (1) or (4) may also be written in the form

$$2W = (\sigma_x u_x+\tau_{xy}u_y+\mu_x\omega_z)_{,x} + (\sigma_y u_y+\tau_{yx}u_x+\mu_y\omega_z)_{,y} \quad . \tag{7}$$

In deriving eq. (7), use has been made of the equations of static equilibrium

$$\sigma_{x,x}+\tau_{yx,y} = 0 \ , \quad \tau_{xy,x}+\sigma_{y,y} = 0 \ , \quad \tau_{xy}-\tau_{yx}+\mu_{x,x}+\mu_{y,y} = 0 \ . \tag{8}$$

Now, the total energy stored in the Cosserat medium may be obtained by integrating eq. (7) and applying the divergence theorem. The result is

$$2\iint_R W \, dA = \int_L [(\sigma_x u_x+\tau_{xy}u_y+\mu_x\omega_z)dy - (\sigma_y u_y+\tau_{yx}u_x+\mu_y\omega_z)dx] \quad . \tag{9}$$

Expressing all quantities in eq. (9) in the directions normal and tangent to the boundary L, eq. (9) becomes

$$2\iint_R W \, dA = \int_L (\sigma_n u_n + \tau_{ns}u_s + \mu_n\omega_z) \, ds \quad . \tag{10}$$

Uniqueness Theorem for Concentrated Loads

Based on the positive definiteness of W and eq. (10), the following

[6] A comma is used to indicate differentiation such as $u_{x,x} = \partial u_x/\partial x$, etc.

uniqueness theorem in the couple-stress theory of elasticity may be established:

Let $\sigma_x^{(1)}$, $\sigma_y^{(1)}$, $\cdots$, $\omega_z^{(1)}$ and $\sigma_x^{(2)}$, $\sigma_y^{(2)}$, $\cdots$, $\omega_z^{(2)}$ be two possible solutions which are continuous and have piecewise continuous first partial derivatives in an open region containing R and its boundary L. Then the difference solution $\Delta\sigma_x = \sigma_x^{(1)} - \sigma_x^{(2)}$, etc. vanishes if and only if

$$\int_L (\Delta\sigma_n \Delta u_n + \Delta\tau_{ns} \Delta u_s + \Delta\mu_n \Delta\omega_z)\, ds = 0 \quad . \qquad (11)$$

The aforementioned theorem can be applied to problems involving singular loads if the points at which stress discontinuities occur are cut out from the region R. The original problem is then recovered by letting the size of the cut vanish. For definiteness sake, let a point O on the boundary L be subjected to concentrated forces p, q and a couple m as shown in Fig. 1(a). Now, consider a small semi-circular identation of radius ρ removed from R and subject it to a system of finite stresses which are statically equivalent to p, q and m as

$$p = \int_{\ell} [\sigma_n^{(i)} \cos\theta - \tau_{ns}^{(i)} \sin\theta]_{r=\rho} \rho\, d\theta, \quad q = \int_{\ell} [\sigma_n^{(i)} \sin\theta + \tau_{ns}^{(i)} \cos\theta]_{r=\rho} \rho\, d\theta \quad ,$$
$$m = \int_{\ell} [\mu_n^{(i)}]_{r=\rho} \rho\, d\theta \qquad (12)$$

where i = 1,2 and ℓ represents the interval $-\pi/2 \leqslant \theta \leqslant \pi/2$. The region R_1 in Fig. 1(b) is defined such that $R_1 \to R$ when ρ approaches zero. The expressions for p, q and m in eq. (12) are required to be integrable in the limit as $\rho \to 0$. Thus, the order of the stress singularities for $\sigma_n^{(i)}$, $\tau_{ns}^{(i)}$ and $\mu_n^{(i)}$ can at most be r^{-1}. Taking the difference of the two possible stress states denoted by

$$r^{-1} f_1(\theta) = \Delta\sigma_n \cos\theta - \Delta\tau_{ns} \sin\theta, \quad r^{-1} f_2(\theta) = \Delta\sigma_n \sin\theta + \Delta\tau_{ns} \cos\theta, \quad r^{-1} f_3(\theta) = \Delta\mu_n,$$

eq. (12) reduces to

$$\int_{\ell} f_i(\theta)\, d\theta = 0 \quad , \quad i = 1,2,3 \qquad (13)$$

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Using eq. (13) and applying eq. (11) to the problem illustrated in Fig. 1(b) render the condition for uniqueness of solution[7]:

$$\int_{\ell} [f_1(\theta)\, g_1(\rho,\theta) + \cdots + f_3(\theta) g_3(\rho,\theta)]\, d\theta = 0 \quad , \qquad (14)$$

in which $g_i(\rho,\theta)$ are continuous functions of ρ, θ and are given by

$$g_1(\rho,\theta)=\Delta u_n \cos\theta - \Delta u_s \sin\theta, \quad g_2(\rho,\theta)=\Delta u_n \sin\theta + \Delta u_s \cos\theta, \quad g_3(\rho,\theta)=\Delta\omega_z \quad .$$

Mean-Value Theorem

To establish eq. (14), the generalized first mean-value theorem of the integral calculus will be employed.

Let $f(x)$ and $g(x)$ be two continuous functions in the interval $a \leqslant x \leqslant b$, where $f(x) \geqslant 0$. There exists a number α intermediate between a and b such that

$$\int_a^b f(x)\, g(x)\, dx = g(\alpha) \int_a^b f(x)\, dx$$

Further, if L represents the union of disjoint intervals on each of which $f(x)$ is always positive (or negative), then the extension of the above theorem is

$$\int_{\ell} f(x)\, g(x)\, dx \leqslant g(\alpha) \int_{\ell} f(x)\, dx \quad , \quad \alpha \in \ell \qquad (15)$$

The notation

$$I_i(\rho) = \int_{\ell} f_i(\theta)\, g_i(\rho,\theta)\, d\theta \quad , \quad i = 1,2,3 \text{ (no sum on i)} \qquad (16)$$

which stands for a typical term of eq. (14), will be adopted. Therefore, it suffices to establish the uniqueness of solution by showing that $I_i(\rho) \to 0$ as $\rho \to 0$. Letting $\ell = \ell_1 + \ell_2$ with the requirements that

[7] The usual boundary conditions are satisfied on $L-\ell$ so that

$$\int_{L-\ell} [\Delta\sigma_n \Delta u_n + \cdots + \Delta\mu_n \Delta\omega_z]\, ds = 0 \quad .$$

$$f_i(\theta) \geqslant 0 \quad \text{on } \ell_1 \text{ , and } \quad f_i(\theta) \leqslant 0 \quad \text{on } \ell_2$$

eq. (13) yields

$$\int_{\ell_1} f_i(\theta)\, d\theta = -\int_{\ell_2} f_i(\theta)\, d\theta = K > 0 \quad . \tag{17}$$

Making use of eqs. (15) and (17), eq. (16) may be put into the form

$$\begin{aligned} I_i(\rho) &= \int_{\ell_1} f_i(\theta) g_i(\rho,\theta) d\theta + \int_{\ell_2} f_i(\theta) g_i(\rho,\theta) d\theta \\ &\leqslant K[g_i(\rho,\theta_1) - g_i(\rho,\theta_2)] \end{aligned} \tag{18}$$

Since $g_i(\rho,\theta)$ is continuous on ℓ, there exists a $\delta_1 > 0$ such that when the distance between the points (ρ,θ_1) and (ρ,θ_2) is less than δ_1, the condition

$$\left| g_i(\rho,\theta_1) - g_i(\rho,\theta_2) \right| < \frac{\varepsilon}{K}$$

holds for every positive number ε. Hence, there is a $\delta > 0$ such that whenever $\rho < \delta$[8],

$$\left| I_i(\rho) \right| < \varepsilon \quad \text{for} \quad \varepsilon > 0$$

and thus

$$\lim_{\rho \to 0} I_i(\rho) = 0 \quad .$$

This completes the proof of the uniqueness theorem for concentrated-load problems in the linear couple-stress theory of elasticity.

Concluding Remarks

The results of the present investigation provide the following conditions for uniqueness:

(1) Conventional- and couple-stresses must be continuous and have piecewise continuous first partial derivatives at every point of the medium except, perhaps, at the point where the concentrated loads are applied. The same conditions must be satisfied by the displacements and rotation. Geometric discontinuities are to be excluded.

[8] For this problem, $\delta = 1/2\ (\delta_1)$.

(2) All quantities such as stresses, displacements, etc. must vanish as $r \to \infty$ if the boundary extends to infinity.

(3) The conventional- and couple-stress singularities can at most be $O(r^{-1})$, where r is the radial distance measured from the point of application of the concentrated loads.

(4) The stress system on a semi-circular cut about $r = 0$ must be statically equivalent to the applied loads.

(5) At other points of the boundary, the usual boundary conditions such as σ_n, τ_{ns} and μ_n must satisfy their prescribed values.

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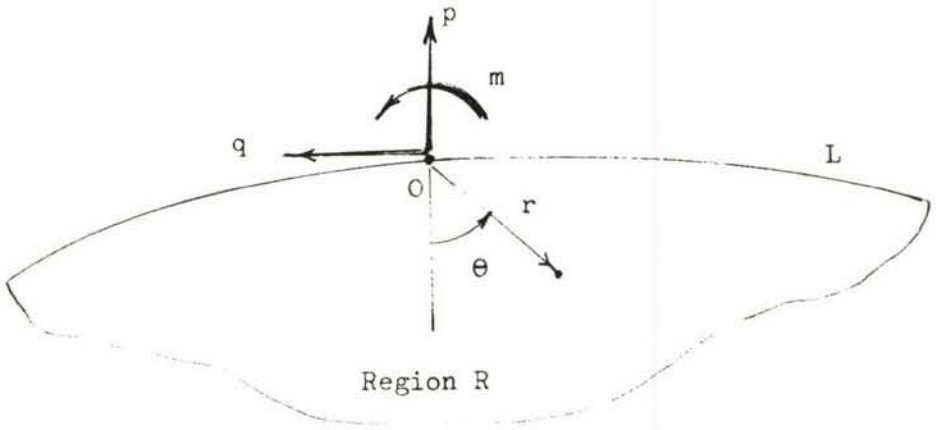

Fig. 1(a) - Concentrated forces and couple.

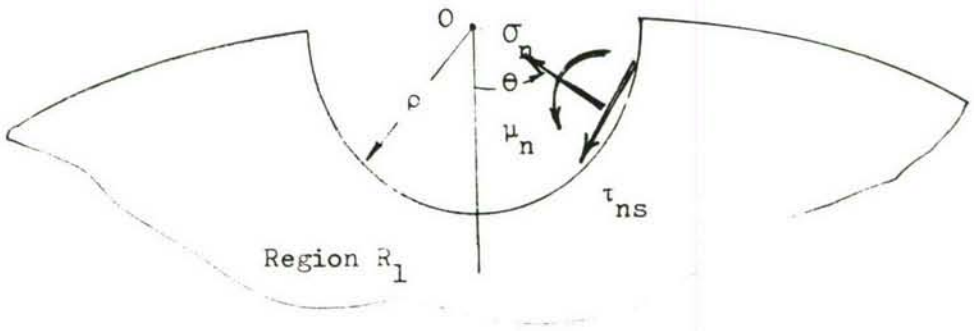

Fig. 1(b) - Equivalent distributed loads.

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